NIKOLA TESLA

AND

THE DEVELOPMENT OF RF POWER SYSTEMS

by

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and

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"Is there, I ask, can there be, a more interesting study than that of alternating currents?"

Nikola Tesla, Vice President of the AIEE[***]

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[***] From Tesla's lectures before the Institution of Electrical Engineers and The Royal Institution, London, England, on February 3 and 4, 1892, published in the Journal of the IEE (London), Vol. XXI, 1892, pg. 51. Reprinted in Experiments with Alternate Currents of High Potential and High Frequency, by Nikola Tesla, McGraw-Hill, 1904, pg. 6.

. . .

The Lumped Element Regime Output Voltage. The lumped element coupled tuned circuit voltage across the capacitor C_2, $v_{C2}(t)$ is now desired. (One should note that C_2 is not a simple physical structure if the distributed capacity of the secondary is appreciable.) This can be found by performing the following integration:

$$v_{c2}(t) = (1/C_2) \int i_2(t)\, dt. \qquad (1)$$

The resultant expression for the voltage across the *lumped* capacitance may be arranged into the convenient form:

$$V_{c2}(t) = F_1 e^{-\alpha_1 t} \cos(\omega_1 t + \gamma_1) + F_2 e^{-\alpha_2 t} \cos(\omega_2 t + \gamma_2) \qquad (2)$$

where the following constants have been introduced

$$F_{1,2} = \frac{-K_{1,2}}{C_2 \sqrt{\alpha_{1,2}^2 + \omega_{1,2}^2}} \qquad (3)$$

$$\gamma_{1,2} = \psi_{1,2} + \theta_{1,2} \qquad (4)$$

$$\theta_{1,2} = \tan^{-1}\left(\frac{-\alpha_{1,2}}{\omega_{1,2}}\right) \qquad (5)$$

and the quantities $K_{1,2}$, $\alpha_{1,2}$, $\omega_{1,2}$, and $\psi_{1,2}$ are all functions of the circuit parameters as defined above.

Finally, we observe that the equations for the currents may be placed in an envelope-and-phase form which is of considerable utility. For example,

$$i_2(t) = R(t) \cos[\omega_1 t + \zeta(t)] \qquad (6)$$

The phase $\zeta(t)$ is a function of the spectral spread $\Delta\omega$, where

$$\Delta\omega = \omega_1 - \omega_2 \qquad (7)$$

The envelope is expressed by

$$R(t) = \sqrt{K_3^2 e^{-2\alpha_1 t} + K_4^2 e^{-2\alpha_2 t} + 2K_3 K_4 e^{-(\alpha_1 + \alpha_2)t} \cos(\Delta\omega t + \psi_1 - \psi_2} \qquad (8)$$

This last expression is extremely useful. *When it is at its peak, the maximum energy can be present in the secondary.* It is at such an instant that one desires the primary spark to break, creating an open circuit (an infinite impedance) in the primary mesh. Such performance by the primary switch would "trap" the maximum electromagnetic energy so that it could only collapse into the secondary.

Spark Durations. As an example of the value of the expression for R(t), one can now determine the instant when the envelope of $i_2(t)$ is a maximum. This instant is designated as the desired primary spark *dwell* t_s. The situation results in a rather complicated equation to solve for t_s. However, the problem is simplified if one considers the relatively low loss case where the exponentials are slowly decaying. [This is a classical adiabatic invariant.] In this situation, taking the derivative, setting it equal to zero and solving for t_s gives the first instant of envelope maximum as

$$t_s = \frac{\psi_2 - \psi_1}{\Delta\omega} \qquad \textbf{(9)}$$

The effect of including losses will reduce the actual value of t_s somewhat. (It is always better to actually calculate R(t) and read off t_s than to use the low-loss rule of thumb.) For a low loss circuit ψ_1 . 0 and ψ_2 . π. Thus, for low losses, the instant at which the envelope of $i_2(t)$ is maximum is approximately t_s . 1/(2Δf). This is half the Abeat period" of the envelope of $i_2(t)$.

Complete Transfer of Energy In the lossless case A = C = 0 in Equation (18), *and* $k_c = 0$. For the situation where the primary and secondary are independently tuned to the same frequency,

$$\frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}} = \omega_o = 2\pi f_o \quad , \qquad \textbf{(10)}$$

Equation (18) becomes

$$(1 - k^2)\,\omega^4 - 2\omega_o^2\,\omega^2 + \omega_o^4 = 0 \qquad \textbf{(11)}$$

with the roots

$$\omega_{1,2} = \frac{\omega_o}{\sqrt{1 \text{ K } k}} \quad . \qquad \textbf{(12)}$$

Skilling[i] has pointed out that the complete transfer of energy from the primary to the secondary occurs at the moment conditions such that the first and second terms of Equation (11) are cancelling while the first and second terms of Equation (12) are adding. From Equations (13) and (14), one will have the equalities

$$K_1 = K_2$$
$$K_3 = K_4 \qquad \textbf{(13)}$$

when $\omega_1 = 2\omega_2$. Further, under this condition Equations (16) and (17) give

$$\phi_1 = -\pi \qquad \phi_2 = 0$$

$$\psi_1 = 0 \qquad \psi_2 = \pi \tag{14}$$

and Equation (37) gives the required coefficient of coupling as

$$k = \frac{3}{5} = 0.6 \tag{15}$$

which is fairly tight. The latter condition was found by Finkelstein, et. al.[ii] and is called “the optimum double resonance principle.” [Total transfer of energy, in the transient, lumped, coupled mode of operation, occurs when the upper normal mode frequency is twice the lower normal mode frequency. For this condition the required coefficient of coupling is k = 0.6.]

Now, this is important if one is to use the optimum principle: From Equation (34), the moment of total energy transfer, for lumped lossless coupled coils, occurs at

$$t_s = \frac{1}{2\Delta f} = \frac{0.632}{\omega_o} \cdot \frac{4}{f_o} \quad . \tag{16}$$

At this instant, Equations (11) and (12) give

$$i_1(t_s) = 0 \tag{17}$$

$$i_2(t_s) = i_{2,\text{Max}} = 2\,K_3 \quad . \tag{18}$$

That is to say, tuning $f_p = f_s$, adjusting the mutual coupling to k = 3/5, *and* breaking the primary spark at t = t_s gives total transfer of energy to the secondary *and traps it there* at time t_s. The primary spark must be broken at this instant (neither before nor after). Most people overlook the switch *duration* requirement associated with k = 0.6 operation. The "optimum" principle is a property of *lumped* coupled operation. Interestingly, the data recorded in Tesla's diary during November of 1899 is

L_1 = 57 µH L_2 = 9 mH M = 425 µH

which plainly shows that his lumped element master oscillator was operating with k = 0.594 . 0.6. Further, the desired spark duration would have been on the order of t_s = 9 µs. (This was a major accomplishment with the technology available in 1899. Space does not permit a discussion of the extensive technology which Tesla disclosed in US patents, prior to the turn of the century, to achieve these switching speeds and spark durations.)

As a side note of practical benefit, if the reader must employ long spark durations, due to the primary switch technology utilized, then loose coupling, on the order of k_c, is probably acceptable. (In fact, critical coupling implies that ♠f 6 0 and t_s 6 4.) The problem with such operation in the transient case, however, is that less than optimum performance is obtained.

In summary, under the ideal conditions described above, the $i_2(t)$ waveform is maximum at the same instant that the envelope of $i_1(t)$ passes through zero. At this instant, all of the energy initially stored in C_p, minus resistive losses, has been transferred to the secondary. Physically, t_s is how long the primary spark duration is desired to last in the primary - *and no longer* - for the energy exchange in the coupled oscillator to go through to completion.

Being able to control the spark depends upon the construction of the rotary break. Rotary breaks quench the spark because the toothed wheel, or spikes, move and draw out the spark until it extinguishes. The ions which are formed between the points quickly recombine to form an insulator. If extremely tight coupling is used, the gap may refuse to quench because the duration of the low current notch in R(t) may be too short to permit sufficient ion recombination in the gap to lower the conductivity. **For Tesla's work, it would have been desirable to have had an electronic switch that would permit control of t_s at the microsecond level, have a reverse breakdown in excess of 100 kV and be able to repetitively switch kiloampere pulses at speeds up to 500,000 per second. Apparently, even at this late date, appropriate high voltage solid state RF technology has not yet developed to the same level of performance as mechanical spark gaps, which can be made to push the limit of capacitor technology.**[iiiivv] **For reasons of economy, we employed the classic rotary spark gap in our research.**

In the linear analysis above we have, of course, neglected the effect of the break speed and only analyzed a *single* discharge of the primary capacitance. This would be acceptable provided the break speed were slow enough so that the RF oscillations ring and die during the break period. Further, the analysis assumes that the secondary electrode has not been permitted to reach breakdown: no air discharges have occurred.

Pulse Interval (Break Speeds). From the considerations above, we can now predict the optimum spark duration. We now ask, AWhat is the best break speed to run?" In the case of a single RLC tuned circuit, it is a straightforward exercise to show that the desirable PRF would be equal to the resonant frequency. (One may pass directly into class C operation this way.) The power developed, or energy flow through the machine, is related to the break speed by

$$P = \frac{dW}{dt} = \frac{1}{2} N C_1 V_o^2 \quad watts \qquad (19)$$

where N is the number of capacitive charges the source provides to C_1 per second. What limits the power delivered now is the KVA rating of the power transformer or prime power generator. Tradeoffs have to be made between the generator's ratings, the desired spark duration and pulse repetition frequency. At Colorado Springs, Tesla frequently operated his primary at 50 kV RMS (V_o = 70.7 kV peak) with about a thousand breaks per second, which corresponds to an average power a little over 300 kW.

The theory presented above provides a very compact analysis in closed form. No numerical approximations have been used in the expressions for $i_1(t)$, $i_2(t)$, or $V_{C2}(t)$. These expressions are exact in the oscillatory case. Before turning our attention to the situation where the distributed nature of the resonator becomes important, we have one last consideration to establish.

The Lumped Element Regime Secondary Induced Voltage. In the circuit equivalent model for coupled coils, the voltage induced into the secondary follows as

$$v_{ind}(t) = M\frac{di_1}{dt} \qquad (20)$$

(Notice that this is **not** the voltage across some Aequivalent" lumped element C_2, which was derived above.) Carrying out the differentiation leads to the expression

$$\begin{aligned} v_{ind}(t) &= MK_1\, e^{-\alpha_1 t}[\omega_1 \cos(\omega_1 t + \phi_1) - \alpha_1 \sin(\omega_1 t + \phi_1)] \\ &\quad + MK_2\, e^{-\alpha_2 t}[\omega_2 \cos(\omega_2 t + \phi_2) - \alpha_2 \sin(\omega_2 t + \phi_2)] \end{aligned} \qquad (21)$$

This is the voltage induced directly into the distributed secondary, and it is a maximum at $t = t_s$, i.e. - when the envelope of $i_1(t)$ is zero. This is the instant when $i_2(t)$ is maximum.

During the spark dwell time ($t \# t_s$) the RF portion of the system may be modeled by lumped coupled circuits. This is because during the primary spark, the primary and secondary are mutually Abathed" in each other's Ainstantaneous" external magnetic fields over their entire physical dimensions. The phase delay *between* the coils is negligible, and they behave as lumped elements for which the classical analysis of coupled coils is appropriate. (The structures are electrically small, their physical dimensions being much, much less than an appreciable portion of a free space wavelength.)

The energy interplays between the primary and secondary, building up to a maximum secondary current at t_s. In this temporal regime, it is perfectly appropriate to determine the currents $i_1(t)$, $i_2(t)$ and voltage $v_{ind}(t)$ from the lumped circuit model. Terman has observed that,

> "The secondary current is exactly the same current that would flow if the induced voltage were applied in series with the secondary and if the primary were absent." [vi]

This equivalence has been extended to the case of distributed lines and resonators by R.W.P. King in a series of classic papers in which he proved that the voltage induced in a distributed transmission line by a lumped primary oscillator can be accurately represented as a point generator, located along the resonator at the center of symmetry of the primary.[viiviii] For a "monopole" resonator above a groundplane, as used in our apparatus, this equivalent point source is located at the base of the secondary.

After the primary spark extinguishes, the secondary slow wave resonator stands alone. The phase delay of current propagation *along* the helix is now substantial.[ix] The *trapped energy* collapses into the resonator and establishes the buildup of a slow wave VSWR pattern, which then rings down and dies out exponentially due to the resonator losses (if there is no discharge, of course). This voltage rise in the secondary, which occurs *after* t_s, is completely overlooked in the lumped element Aoptimum performance" analysis presented above.

Trapping the Energy in the Secondary. If the spark gap is open, then the primary will appear as an "infinite" series impedance to any back EMF induced across the primary turns by $i_2(t)$. If the primary spark is "blown out" when the primary current is zero, all of the magnetic energy must be *trapped* in the secondary. As we have pointed out in several publications,[xxi] this fact was apparently first disclosed by Fleming and Dyke, and discussed by W.H. Eccles, in 1911.[xii] Fleming and Dyke obtained the peak of the Fourier spectrum of the secondary with a wavemeter. The spectra which they record shows three humps: Two displaced humps, with a large distinct hump in the center of the spectrum. If they tightened the coupling, the outer humps spread further apart. If they loosened the coupling, the outer humps approached one another and, near critical coupling, all three coalesced into one. Fleming and Dyke noted that,

> "The frequency corresponding to the middle hump is the natural free period frequency of the secondary circuit. The frequencies of the other two maxima on either side correspond to the two oscillations which are created by the reaction between the primary and secondary circuits."

Eccles agreed, further observing that,

> "The fact that Dr. Fleming's curves show, for some degrees of coupling three humps instead of two, indicates that the pair of circuits **does not remain a double system throughout the oscillation**. One circuit disappears at a more or less early stage of the process, that is to say, the primary spark goes out and virtually removes the primary circuit from the combination, so that **thereafter the secondary circuit vibrates alone**."

We show the coupled oscillations (prior to t_s), the primary break (at t_s), and the subsequent passage to distributed resonator mode in the time domain, as well as a three humped spectrum (double hump prior to t_s and single hump after t_s) that a wave meter would detect in Figure 3. Remember that a wave meter was a primitive instrument used for spectrum analysis in the early days of wireless, and its response was too slow to reveal that the double-hump and peaked single-hump were occurring at different times. It has been our hypothesis that during the later time, when "the secondary circuit vibrates alone," it can behave as a *slow wave* helical resonator and that ***voltage rise can be by standing waves***. We have observed that the VSWR voltage build-up can easily be *an order of magnitude greater* than the lumped element voltage rise. Figure 4 shows an actual oscilloscope measured voltage rise, by a factor of 18, after the primary spark breaks. Several beat periods can be seen during the spark duration. Notice, particularly, that the measured coherence time (or wave interference build-up time after the spark break) is approximately 22.4 µs. (The theory presented below (see Equation (65)) predicts a coherence time of 20.42 µs for this coil, which resonated at 122 kHz and had a Q of 31.3.)

Today, a similar phenomenon is observed in UWB (Ultra-Wideband) impulse radar. When a target (a long cylindrical structure, for example) is illuminated by a short duration pulse, the radar return consists of two components - an "early time" response term and a "late time" (or sum of exponentials) term.[xiii] The first corresponds to the response of the target while the RF excitation pulse (of finite duration and spatial extent) is sweeping over the target and is "forcing" a response from the "distributed circuit." During this time-interval surface excited traveling waves are shedding radiation as they propagate (i.e. the structure is behaving like a traveling wave antenna), and standing waves are also beginning to form. The structure then starts to radiate energy like a resonant dipole antenna. After the "short pulse" has passed by, the target structure, upon which standing waves have been established, rings down at its own characteristic frequencies. We now turn to an analysis of Tesla's wonderful resonator itself.

The Helical Resonator

Perhaps the most satisfactory description of voltage rise on a helical resonator, from an engineering perspective, is that given in the references.[xiv][xv][xvi][xvii] The analysis points out that the secondary of a Tesla transformer is, in fact, a quarter wave helical resonator with a high VSWR. (Unlike so many other goals in RF engineering, the higher the VSWR on a resonance transformer the better.) In the sinusoidal steady state, the Vmax at the top of the resonator is simply the product of the VSWR and the voltage injected into the base of the resonator (the place of the V_{min}). To the best of our knowledge, there exists no useful exact solution for the *transient* fields of a helically wound slow wave resonator. (The transient response of uniform electrical resonators, such as aircraft, is currently an area of prominent research activity, but no structures with slow wave surfaces have been analyzed in the open literature, that we know of.) Because resonance transformers have such high Q (narrow bandwidth), the steady state, quasi-monochromatic analysis given below is pragmatically justified and proves itself to be remarkably accurate for engineering purposes.[*]

Voltage Magnification by Standing Waves. The voltage "magnification" process is most easily understood by considering the voltage rise on a generic transmission line,[xviii] as shown in Figure 5. As usual, the coordinate origin is taken at the load and (for pedagogical reasons) a time harmonic generator is assumed to drive the input end at z = - P. (We know that the spark switched sources employed are not truly monochromatic, but the sinusoidal approximation is appropriate for adiabatic resonator oscillations.) The voltage at any point along the line is a solution of the wave equation, and is given by the expression

$$V(z) = V_+ e^{-\gamma z} + V_- e^{+\gamma z} \qquad (22)$$

where z = 0 at the load and z = -P at the generator end. (We have implicitly assumed $e^{j\omega t}$ time harmonic field variation.) Physically, Equation (47) expresses the fact that the voltage at any point along the transmission line is the superposition of a forward travelling wave and a backward travelling wave. The resultant analytical expression describes a spatially distributed interference pattern called a standing wave. As usual, **γ** is the complex propagation constant $\gamma = \alpha + j\beta$. The complex constants V_+ and V_- follow from the second order partial differential equation of which Equation (47) is a solution (the "transmission line equation"), and depend upon the boundary conditions (the generator and the load).

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[*]All electrical engineering undergraduates will recall that transient waveforms are *finite energy signals*, while periodic waveforms are *finite power signals*. (See any signals and systems text, for example, Continuous and Discrete Signals and System Analysis, by C.D. McGillem and G.R. Cooper, Holt, Rinehart and Winston, 1974, pp. 34-35, 150-151.) We introduce this distinction while passing to the steady state model of the distributed helical resonator, and assert that the analytical regime is of pragmatic value since the coherence time of waves on the resonator is so long, i.e. the canonical variables are, essentially, adiabatic invariants. (See, for example, Mechanics, by L.D. Landau and E.M. Lifshitz, Pergamon Press, third edition, 1976, pp. 154-159.)

Also, at the load end it is customary to define the complex load reflection coefficient **Γ**:

$$\Gamma_2 \times \frac{V_-}{V_+} = \frac{Z_L - Z_0}{Z_L + Z_0} = *\Gamma_2* (\phi_2. \qquad \textbf{(23)}$$

where Z_L is the impedance of the load and Z_o is the effective impedance of the line. For a capacitive load, $Z_L = 1/(j\omega C)$, and for an open circuited line **Γ** = 1 $|\underline{0E}$. Tesla's data for these quantities can readily be plotted on a Smith chart, from which it is visually obvious that he was operating in the quarter-wave resonant condition.

From Equation (47) we have, at the input end of the line,

$$V_{input} = V(-liter) = V_+ e^{\gamma liter} + V_- e^{-\gamma liter} \qquad \textbf{(24)}$$

where, again, **Γ** is a complex quantity defined at the load. Also from Equation (47), we may write the voltage at the load end as:

$$V_{Load} = V(0) = V_+ + V_- = V_+ [1 + \Gamma] \textbf{ (50)}$$

Equations (49) and (50) may be combined in the following extremely useful expression which relates the load voltage to the input (generator end) voltage:

$$V = \frac{V_{base} [1 + \Gamma_2]}{[e^{\gamma liter} + \Gamma_2 e^{-\gamma liter}]} \qquad \textbf{(25)}$$

This expression is probably the most important equation in all of high voltage RF engineering. (We propose to call it "The Tesla Equation," since he was the first to patent apparatus utilizing the phenomenon that it describes.) For computational purposes the magnitude may be expanded as the expression

Error! **(26)**

Now consider what happens on an open-circuited low-loss line one quarter wavelength long. Simple complex algebra gives the following well-known result:

$$V_{Load} \$ -j\left(\frac{V_{base}}{\alpha liter}\right) \quad \textbf{(27)}$$

(for P = n**λ**/4 with n odd) where, again, **α** is the attenuation per unit length of the transmission line, and the j implies that the voltages at the two ends are in phase quadrature. The structure is a lossy, tuned reactive resonator. *Since the numerator of this equation is finite and the denominator is vanishingly small, the voltage standing wave will build up to very large values.* The transient growth process will evolve until either a system nonlinearity occurs (breakdown) or the line's I5R losses are equal to the power being supplied by the source. The power driving the line, the line losses αP, and the breakdown potential of the load geometry (which usually arises either from free electron avalanche or from the onset of cold field emission from the electrode) are what limits the maximum voltage ultimately attainable with this apparatus. Again, we have observed that practical voltage rise by VSWR can easily be an order of magnitude greater than lumped element voltage rise.

In order to carry out a numerical evaluation of this analysis, it is necessary to calculate the real and imaginary parts of the complex propagation constant and Schellkunoff's effective characteristic impedance for a helical resonator.[xix] These can be found from the relations

$$\alpha liter = \frac{Rliter}{2Z_o} = \frac{R_{loss}}{2Z_o} \qquad \textbf{(28)}$$

$$\beta = 2\pi ª \lambda_g \quad \text{where} \quad \lambda_g = V_f \lambda_o \qquad \textbf{(29)}$$

where the helix velocity factor given in our publications is expressed as

$$V_f = \frac{v}{c} = \frac{1}{\sqrt{1 + 20(Dªs)^{2.5}\,(Dª\lambda_o)^{0.5}}}$$

λ_o = free space wavelength **(30)**

D = helix diameter

s = turn - - turn spacing

(all the same units)

which is explained in the Appendix below. The effective characteristic impedance (those unfamiliar with this term should consult Jordan and Balmain[xx]), and the attenuation, are found from

$$Z_o = \frac{60}{V_f}\left[\ln\,(4HªD) - 1\right] \qquad \textbf{(31)}$$

and

$$\alpha liter = \frac{7.8125(HªD)^{1ª5}}{d_w Z_o \sqrt{f_{MHz}}} \; Nepers$$

where

(32)

d_W = wire diameter inches

H = Ns = height of helix

N = Number of turns

D = Helix diameter

At this point, the engineering analysis is complete, and we are now ready to perform actual numerical predictions to compare with system measurements.

Summary of Operation

The secondary of a conventional resonance transformer is a helically distributed quarter-wave resonator, *not* a lumped tuned circuit - at least when it is operating in the regime where the primary has been open circuited. The voltage rise is by standing wave: $V_{max} = S\ V_{min}$, where S is the VSWR on the transmission line resonator. The actual measured voltage distribution on a transmission line resonator essentially follows the first ninety degrees of a *spatial* sinusoid, much as it would on a quarter-wave vertical monopole antenna: V_{min} at the base and V_{max} at the top. In fact, resonance transformer engineering is exactly the same as designing top loaded vertical antennas. The only distinction is that a Tesla coil is *physically* short enough so that its radiation resistance is negligible. (In the absence of breakdown, very little energy is actually lost by radiation.)

These Tesla resonators perform equally well when driven by any high power *master oscillator* - spark gap, vacuum tube or solid state. (Efficiency and junction breakdown become major concerns with the latter two.) This Tesla coil theory has been used to build high voltage structures well up into the microwave region, and not just at VLF.

REFERENCES

i. Skilling, H.H., Transient Electric Circuits, McGraw-Hill, 1952, pp. 231-233.

ii. Finkelstein, D., P. Goldberg, and J. Shuchatowitz, "High Voltage Impulse System," The Review of Scientific Instruments, February, 1966, pp. 159-162.

iii. Grekhov, I., "New Principles of High Power Switching with Semiconductor Devices," Solid-State Electronics, Vol. 32, No. 11, 1989, 923-930.

iv. Tuchkevich, V.M., and I.V. Grekhov, Novye Printsipy Kommutatsii Bol'shikh Moshchnostej Poluprovodnikovymi Priborami (New Principles for Switching High Power Using Semiconductor Devices), Leningrad, Nauka Press, 1988.

v. Astrova, E.V., "Ultrahigh Voltage Silicon P-N Junctions with a Breakdown Voltage above 20 kV," Solid-State Electronics, Vol. 32, No. 11, 1989, pp. 851-852.

vi. Terman, F.E., Electronic and Radio Engineering, McGraw-Hill, 4th edition, 1955, Pg. 58.

vii. King, R.W.P., "The Application of Low-Frequency Circuit Analysis to the Problem of Distributed Coupling in Ultra-High-Frequency Circuits," Proc. I.R.E., November, 1939, pp. 715-724.

viii. King, R.W.P., "General Amplitude Relations for Transmission Lines with Unrestricted Line Parameters, Terminal Impedances, and Driving Point," Proc. I.R.E., December, 1941, pp. 640-648.

ix. Ramo, S., J.R. Whinnery and T. Van Duzer, Fields and Waves in Communication Electronics, Wiley, 1984, pg. 209. (See Problem #4.10b.)

x. Corum, K.L., and J.F. Corum, TCTUTOR, loc cit. (See Appendix IV.)

xi. Corum, K.L., and J.F. Corum, "Tesla Coils: 1890-1990 - 100 Years Of Cavity Resonator Development," Proceedings of the Fourth International Tesla Symposium, Colorado Springs, Colorado, July, 1990.

xii. Fleming, J.A., and G.D. Dyke, "Some Resonance Curves Taken With Impact and Spark Ball Dischargers," Proceedings of the Physical Society, London, Vol. 23, February, 1911, pp. 136-146. [Discusses spectral response of double tuned lumped coupled circuits as a function of coupling for quenched and unquenched spark gap oscillators.]

xiii. Larry, T.L., and M. Van Blaricum, "Transient Scattering from Bodies Designed with Loads or Layers," Proceedings of the First Los Alamos Ultra Wideband Radar Symposium, March 7, 1990.

xiv. Corum, J.F., and Kenneth L. Corum, "A Technical Analysis of the Extra Coil As A Slow Wave Helical Resonator," Proceedings of the 2nd International Tesla Symposium, Colorado Springs, Colorado, 1986.

xv. Corum, J.F., and K.L. Corum, "The Application of Transmission Line Resonators to High Voltage RF Power Processing: History, Analysis, and Experiment," Proceedings of the 19th Southeastern Symposium on System Theory, Clemson University, Clemson, South Carolina, March, 1987, pp. 45-49.

xvi. Corum, J.F., and K.L. Corum, "Tesla Coils: 1890-1990 - 100 Years Of Cavity Resonator Development," Proceedings of the Fourth International Tesla Symposium, Colorado Springs, Colorado, July, 1990.

xvii. Corum, J.F., and K.L. Corum, Vacuum Tube Tesla Coils - Published by CPG Communications, Inc., 1988, (160 pages). [ISBN 0-924758-00-7].

xviii. Terman, F.E., “Resonant Lines in Radio Circuits,” Electrical Engineering, July, 1934, pp. 1046-1053.

xix. Corum, J.F., and K.L. Corum, “The Application of Transmission Line Resonators To High Voltage RF Power Processing: History, Analysis and Experiment,” Proceedings of the 19th Southeastern Symposium on System Theory, Clemson University, 1987, pp. 45-49.

xx. Jordan, E.C., and K.G. Balmain, Electromagnetic Waves and Radiating Systems, Prentice-Hall, 2nd edition, 1968, pp. 385-396.