

**Goubau, G., “Über die Zennecksche Bodenwelle,” (On the Zenneck Surface Wave),
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1. Introduction^a

In 1907 the well-known paper “Über die Fortpflanzung ebener elektromagnetischer Wellen längs einer ebenen Leiterfläche und ihre Beziehung zur drahtlosen Telegraphie,” (On the propagation of plane electromagnetic waves along a flat conducting surface and their relation to wireless telegraphy) by Zenneck appeared.[1]¹ To be sure, Zenneck is recognized as the first to show that the propagation of the long waves employed at that time can not be described by elementary optical laws, since the propagation is not in free space, but rather takes place on the boundary surface between two media. In order to present insight into their behavior in the simplest possible manner, he considered plane waves propagating along the boundary surface between two different media.[2]^{b,2} He found that the structure of the fields and the propagation attenuation depends on the earth’s constants (constitutive parameters). In spite of the simplifying assumptions made, the analysis gave calculated wave polarizations on the earth in good agreement with measurements.

Arnold Sommerfeld (1909) [3]³ was the first to completely solve the problem of the propagation from a dipole over a plane earth, which led to a division of the total field into a “space wave” and a “surface wave”, where the latter exhibits a cylindrical Zenneck wave. This decomposition arises informally from the evaluation of a complex integral for the Hertz vector of the total field. A portion of this integral delivers a pole and the corresponding field component is described as a surface wave. Ten years later, H. Weyl [4]⁴ treated a problem equivalent to Sommerfeld’s, except by another method. In his solution the Zenneck wave does not emerge as an ingredient of the field, and therefore Weyl considered the Sommerfeld decomposition of the total field into space and surface waves as arbitrary, and he disputed the physical meaning of the Zenneck wave.

Since the appearance of Weyl’s publication, recent papers have appeared [5]⁵ that analyze the question of the existence, or, respectively, the nonexistence, of the surface wave. Referring to field measurements would provide evidence for the correctness of the view of the author, as well as, from the outset, asserting that the field near to the ground can never exactly correspond to the Zenneck wave since it does not describe the total field. Also, the space wave in the ground must possess a field component, and the structure of this field must be different than the Zenneck wave.

Doubtlessly, one could have a separate opinion whether it is appropriate to assume a space component and a ground wave, in the consideration of radiation from a dipole, since ultimately one is interested in the total field. In spite of that, the Zenneck wave is of independent historical interest. As will be seen in the present paper, the reader will be presented with a physically realizable wave type, which further leads to extensive analogies with well-known types of waves, for example as the Lecher wave or Sommerfeld’s single wire wave shows. There exists an analogous connection between surface waves and space waves, like that between guided waves and “supplementary fields” simultaneously excited. The supplementary field preserves the continuity of the total field in the vicinity of the excitation source and depends on its structure and condition. In the case of a closed waveguide (concentric conductors, pipe wave guide) it is a purely reactive field, while in the case of an open waveguide it is a radiating field. Whether the

^a In this paper the vectors are denoted by Latin symbols and the vector product is designated by \times .

^b Planar boundary strata-waves were first investigated by K. Uller.²

guided wave field or the supplementary field predominates depends entirely upon the manner of excitation and the sort of observation.

In the case of the symmetrical excitation of a two-conductor transmission line, e.g. by a concentrated power source, the field in the vicinity of the conductors at a short distance from the source is specified, almost entirely, by the Lecher wave. But, only the supplementary field is observed at great distances from the conductors. In the case of very long conductors, the supplementary field can predominate in the immediate region of the conductors; that is the exponentially damped Lecher wave dies away to a sufficiently small amplitude. This is the case due to a very long two-wire conductor; consequently the transmitted power is not from the Lecher wave but from the supplementary field propagating nearby in free space. The beginning and end of the conductor work essentially as transmitting and receiving antennas, and the intervening part of the conductor is barely concerned with the energy transmission.

In the case of a single conductor, if one doesn't take special precautions [6]⁶, one excites almost only the supplementary field, namely the well-known long wire wave, which will be absorbed in the case of long wire antennas. Non-radiating conductor waves (Sommerfeld's wire-waves) would never appear if the conductivity of the wires were infinitely large.

The radiation from a dipole or from other radiators above a plane earth can be understood as the excitation problem of an open waveguide and, to be sure, of a planar waveguide. Therefore, the Zenneck wave exhibits a guided wave-type and the space wave the supplementary field. Since, in the case of conventional antennas, the excitation requirements are unfavorable for guided waves, and, moreover, since their attenuation is generally great, the space-wave plays a deciding role for the total field.

In the following exposition the realizability of the Zenneck wave is discussed first. It is shown that the wave exists if the media possesses infinite conductivity. Furthermore, orthogonality relations between space-waves and surface-waves^c are derived from the fact that the amplitude of the surface wave permits calculation in a simple manner.

2. Realizability of the Zenneck Wave

We will now turn to the question: "Theoretically, what radiation would a Zenneck wave alone [that is, without a supplementary field (space wave)] excite?"

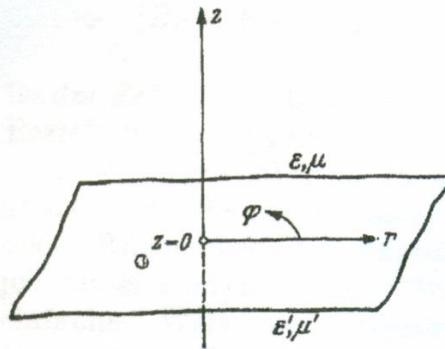


Figure 1. The coordinate system employed.

First of all, we write the field equations of a wave in cylindrical coordinates under the assumption that the media interface is at $z = 0$ and the origin of the wave is at $r = 0$. (See Fig. 1.)

^c Since, in the literature, the phrase "ground wave" is also employed for the *total field on the ground*, let it be explicitly pointed out that, here, the term "surface wave" will always be employed in the Sommerfeld sense for the Zenneck wave.

For $z > 0$

$$\begin{aligned} E_r &= A H_1^{(2)}(\gamma r) e^{j(\omega t - hz)} \\ E_z &= -j \left(\frac{A\gamma}{h} \right) H_0^{(2)}(\gamma r) e^{j(\omega t - hz)} \\ H_{2\phi} &= A \frac{k}{h} \sqrt{\frac{\varepsilon}{\mu}} H_1^{(2)}(\gamma \rho) e^{j(\omega t - hz)} \end{aligned}$$

(1)

For $z < 0$

$$\begin{aligned} E_r &= A H_1^{(2)}(\gamma r) e^{j(\omega t - h'z)} \\ E_z &= -j \left(\frac{A\gamma}{h'} \right) H_0^{(2)}(\gamma r) e^{j(\omega t - h'z)} \\ H_{2\phi} &= A \frac{k'}{h'} \sqrt{\frac{\varepsilon'}{\mu'}} H_1^{(2)}(\gamma \rho) e^{j(\omega t - h'z)} \end{aligned}$$

where

$$\begin{aligned} k &= \omega \sqrt{\mu \varepsilon} \\ k' &= \omega \sqrt{\mu' \varepsilon'} \\ h^2 &= k^2 - \gamma^2 \\ h'^2 &= k'^2 - \gamma'^2 \\ \frac{h}{\varepsilon} &= h' \varepsilon' \\ \left(\frac{1}{\gamma^2} = \frac{1}{k^2} + \frac{1}{k'^2} \quad \text{for } \mu = \mu' \right) \end{aligned}$$

A is a constant. ε , ε' and μ , μ' are the absolute dielectric constants and permeabilities of the two media. (Conductivity should be considered in the complex dielectric constants.) H_0 and H_1 are Hankel functions of orders 0 and 1, and, indeed, the same as those that vanish ($H^{(1)}$ for $Im(\gamma) > 0$, $H^{(2)}$ for $Im(\gamma) < 0$) for the selected root of γ as $r \rightarrow \infty$. The real parts of h and h' are both negative. The values with negative imaginary parts are taken for h , and with positive imaginary parts for h' .

In the immediate region of the z -axis ($r = a$ where $|\gamma a| \ll 1$) one may express the Hankel functions by their small argument approximations, and then obtain

$$H_\phi(a) = \left. \begin{aligned} & -\frac{j}{2\pi} A \sqrt{\frac{\varepsilon}{\mu}} \frac{k}{a\gamma h} e^{j(\omega t - hz)} \quad \text{for } z > 0 \\ & -\frac{j}{2\pi} A \sqrt{\frac{\varepsilon'}{\mu'}} \frac{k'}{a\gamma h'} e^{j(\omega t - h'z)} \quad \text{for } z < 0 \end{aligned} \right\} \quad (2)$$

for the magnetic field components. Since an electromagnetic field is unambiguously specified by its electric or magnetic field components tangent to a closed surface surrounding the sources, the waves under consideration can be thought to be produced by the following forced current distribution on an infinitely long wire:

$$\left. \begin{aligned}
I(z) &= \left. \begin{aligned}
I_o e^{j(\omega t - hz)} & \text{ for } z > 0 \\
I_o e^{j(\omega t - h'z)} & \text{ for } z < 0
\end{aligned} \right\} \\
\text{with } I_o &= -4jA \sqrt{\frac{\varepsilon}{\mu}} \frac{k}{\gamma h} = -4jA \sqrt{\frac{\varepsilon'}{\mu'}} \frac{k'}{\gamma h'}
\end{aligned} \right\} \quad (3)$$

At first, the assumption of an infinitely long conductor with an EMF uniformly impressed over its length appears non-physical. However, an analogous assumption may be implied if one assumes any kind of open waveguide, for example a two-conductor transmission line, and takes only the regular field and neglects the supplementary field. In order to actually obtain such a condition one must force an infinitely extending current component on a metal wall perpendicular to the waveguide.

If one replaces the infinitely long wire by one of finite length, stretching from $z = z_2$ (< 0) to $z = z_1$ (> 0), and forces a current component on it, as described by Eq. (3), over the interval z_2 to z_1 , then one may interpret the resulting field as the superposition of two fields: the infinitely expanding field of a current component reaching from $z = -\infty$ to $z = +\infty$, minus the field emanating from the current path that would run from z_1 to $+\infty$ and z_2 to $-\infty$. If one of the media possesses conductivity, the current component of Eq. (3) decays exponentially for $z \rightarrow +\infty$ as well as for $z \rightarrow -\infty$. The second field component will then be more neglectable the greater z_1 and $|z_2|$ are. The segmented current elements give a radiating space-wave in addition to the obvious surface wave component, and these radiating space waves must be the negative of the space wave that emanates from the current path z_2, z_1 . However, in the case of finite length current paths the space radiation overtakes the surface wave at great distances ($\gamma r \gg 1$) because the field from the current path z_1 to $+\infty$ can propagate through space almost freely, while the surface waves corresponding to Hankel functions of complex argument, which exist over the earth, will tend to decay exponentially. As was already pointed out in the introduction, an analogous phenomenon appears in all open waveguides.

Further, for the physical existence of a field it is essential that the energy flow passing through any kind of surface, finite or infinite, remains finite. One can readily convince himself of the fact that the Zenneck wave satisfies this requirement as long as $a > 0$ and I_o is finite, assuming ε or ε' is complex. Otherwise the field would not decay exponentially as $\pm z \rightarrow \infty$.

By this it is systematically demonstrated that the Zenneck wave is just as realizable as any other treated wave.

3. Orthogonality Between Surface and Space Waves, and Calculation of the Surface Wave Amplitude.

We consider the total fields E_D, H_D of any vertical electric dipole that is elevated at distance $z = z_o$ above or below the surface of the earth (Fig. 2). Let an ideal Zenneck wave source be situated at a distance $r = d$, as was treated in the previous segment discussion, however the radius a of the cylindrical current sheath should be arbitrarily small, that is – the excitation source should shrink to a current filament.

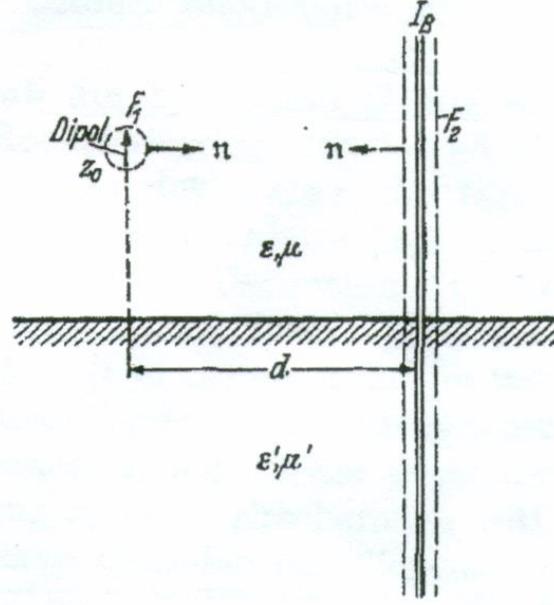


Fig. 2. Dipole and source for Ze neck waves to demonstrate orthogonality.

In this case, if the total current strength in every source-segment remains invariable the field strength at any distance from the source also remains alike. The field of the Ze neck wave for a certain normalized sheath-current strength may be designated by E_B, H_B . Let the frequency be the same as that of the dipole.

We now apply the Reciprocity Theorem to both of the fields E_D, H_D and E_B, H_B . The region of integration extends over the entire enclosing space taken as a sphere (F_1) for the dipole and a cylinder (F_2) surrounding the excitation source of the Ze neck wave. One then obtains:

$$\int_{F_1} \{E_D \times H_B - E_B \times H_D\} n df + \int_{F_2} \{E_D \times H_B - E_B \times H_D\} n df = 0 \quad (4)$$

Since the factor $e^{j\omega t}$ cancels out here, it should be suppressed in all the following relations.

The integral over the boundary surface at infinity vanishes since the surface wave varies exponentially in all directions (ϵ' is always complex). The integral over F_1 can be evaluated in a simple manner. Letting the moment of the dipole be P , one obtains

$$\int_{F_1} \{...\} n df = j\omega P \bullet E_{B_z}(d, z_0) \quad (5)$$

where $E_{B_z}(d, z_0)$ means the z -component of the surface wave on the spot of the dipole. If one lets $A = A_0$ be the amplitude factor in Equation (1), corresponding to a uniformly distributed current, then one obtains

$$\int_{F_1} \{...\} n df = \begin{cases} A_0 \omega P \frac{\gamma}{h} H_0(\gamma d) e^{-h\gamma z_0} & \text{for } z_0 > 0 \\ A_0 \omega P \frac{\gamma}{h'} H_0(\gamma d) e^{-h'\gamma z_0} & \text{for } z_0 < 0 \end{cases} \quad (6)$$

Since the radius a of the Zenneck wave source can be taken as arbitrarily small, the portion of the vector product ($\mathbf{E}_B \times \mathbf{H}_D$) vanishes in the upper surface integral over F_2 in Equation (4). Therefore one has

$$\int_{F_2} \{ \dots \} n df = \int_{F_2} \{ E_D \times H_B \} n df = - \int_{-\infty}^{+\infty} E_D I_{B_z} dz \quad (7)$$

where E_{D_z} is the z-component of the dipole field at the spot of the surface wave source and $I_B(z)$ is the current strength in the source.

We will now assume that the dipole field contains a fraction pE_B , pH_B of a surface wave, where p is a perturbation factor by which the amplitude differs from the basic wave. Therefore, we set

$$E_D = p E_B + E_R, \quad H_D = p H_B + H_R$$

Then E_R , H_R is the space wave field.

Plugging these into Equation (7) gives:

$$\int_{F_2} \{ \dots \} n df = - \int_{-\infty}^{+\infty} E_{R_z} I_B dz - \int_{-\infty}^{+\infty} E_{B_z} I_B dz \quad (8)$$

With these values of E_{R_z} and I_B one obtains from Equations (1) and (3)

$$\begin{aligned} \int_{-\infty}^{+\infty} E_{B_z} I_B dz &= \\ &= -4A_o^2 \sqrt{\frac{\varepsilon}{\mu}} \frac{k}{h} H_0(\gamma d) \left\{ \frac{1}{h'} \int_{-\infty}^0 e^{-2jh'z} dz + \frac{1}{h} \int_0^{\infty} e^{-2jhz} dz \right\} \\ &= 2jA_o^2 \sqrt{\frac{\varepsilon}{\mu}} \frac{k}{h} H_0(\gamma d) \left(\frac{1}{h^2} - \frac{1}{h'^2} \right) \end{aligned} \quad (9)$$

If one introduces Equations (6)-(9) into Equation (4), one obtains

for $z_o > 0$:

$$\begin{aligned} \omega P e^{-jhz_o} - 2jpA_o \frac{k}{\gamma} \left(\frac{1}{h^2} - \frac{1}{h'^2} \right) \sqrt{\frac{\varepsilon}{\mu}} \\ = \frac{1}{A_o H_o(\gamma d)} \frac{h}{\gamma} \int_{-\infty}^{+\infty} E_{R_z} I_B dz \end{aligned} \quad (10a)$$

for $z_o < 0$:

$$\begin{aligned} \omega P e^{-jh'z_o} - 2jpA_o \frac{k'}{\gamma} \left(\frac{1}{h^2} - \frac{1}{h'^2} \right) \sqrt{\frac{\varepsilon'}{\mu'}} \\ = \frac{1}{A_o H_o(\gamma d)} \frac{h'}{\gamma} \int_{-\infty}^{+\infty} E_{R_z} I_B dz \end{aligned} \quad (10b)$$

The left side of these equations contains no d , while the right sides stand as functions of d . That is, $H_0(\gamma d)$, E_{R_z} . Therefore, it must be that

$$\frac{1}{H_0(\gamma d)} \int_{-\infty}^{+\infty} E_{R_z} I_B dz = C \quad (11)$$

where C is a constant independent of d , which one may set equal to zero; if it was not zero then one could still subtract from E so great a surface wave that C would be zero for the remaining space wave. Therefore, if

$$pA_o = \begin{cases} -\frac{j}{2} \omega P \sqrt{\frac{\mu}{\varepsilon}} \frac{\gamma}{k} \frac{h^2 h'^2}{h'^2 - h^2} e^{-jh z_o} & \text{for } z_o > 0 \\ -\frac{j}{2} \omega P \sqrt{\frac{\mu'}{\varepsilon'}} \frac{\gamma}{k'} \frac{h^2 h'^2}{h'^2 - h^2} e^{-jh' z_o} & \text{for } z_o < 0 \end{cases} \quad (12)$$

is extracted from the dipole radiation or the surface wave component, then the orthogonality relation

$$\int_{-\infty}^{+\infty} E_{R_z} I_B dz = 0 \quad (13)$$

is valid for the remaining space wave. Since the z -directed I_B has the same $H\phi$ component as the surface wave, one may also write Equation (13) in the form:

$$\int_{-\infty}^{+\infty} (E_R \times H_B) \tau dz = 0 \quad (14)$$

where τ is any unit vector perpendicular to the z -axis.

The stipulation that the constant C should be null still requires further justification. According to Equation (11), C can be a function of the dipole coordinate z_o . The difficulty of our problem basically lays in the failure of the definition of a surface-wave-free space-wave field. No information about the eventual content of a surface wave component can be obtained from the asymptotic ($r \rightarrow \infty$) behavior of a field since it does not appear because of the exponential damping of such. However, for more basic reasons it obviously follows that Equations (13) and (14), respectively, which follow from the assumption that $C = 0$, bring into play the definition of a surface-wave-free space-wave. First, an analogous orthogonality relation exists for all wire-waves between traveling waves and the supplementary field. [7]⁷ Moreover, (13) follows without constraint if one assumes an “idealized” earth, which is able to lead to an undamped surface wave. One would obtain such if one could supply an infinitely extended perfectly conducting metal planar boundary with a lossless dielectric coating. Above the primary layer the surface wave that would emanate from that configuration differs in its field behavior from a Zenneck wave merely in that the parameter γ in Equation (1) is real, while for the latter it is complex since the Poynting-vector possesses a power-flow component directed into the earth. If the field of a dipole above such an idealized earth passes, in the upper region, asymptotically to the field of the limiting layer, then the amplitude of the undamped primary wave diminishes as $1/\sqrt{r}$, while that of the spatial radiation corresponding to the spatial propagation of energy dies out nearly as $1/r$. If

one employs the reciprocity relations of these components in the case of undamped limiting layer waves, then one immediately discerns that, in the corresponding Equation (10) (which differs only unessentially from that derived here),

$$\frac{1}{H_o(\gamma d)} \int_{-\infty}^{+\infty} E_{R_z} I_B dz = C \quad \text{for} \quad d \rightarrow \infty$$

will vanish since, as mentioned, the strength of E_R varies as the Hankel function with a real argument. Since C is independent of d , the integral $\int E_{R_z} I_B dz$ vanishes for every d .

Since one can interpret the current stream I_B , which produced a pure Zenneck wave, as a dipole series, the surface wave component of each individual dipole must, when summed together, result in the correct amplitude A_o . One can easily convince himself that this leads to the correct amplitudes as calculated by Equation (12).

Let it be further stated that we have

$$\int_{-\infty}^{+\infty} (E_B \times H_R)_t dz = 0 \tag{15}$$

in addition to the orthogonality relation (14). Further, the surface wave amplitude, specified by Equation (12), was derived somewhat differently from that of Sommerfeld; that is, Sommerfeld includes a space wave in addition to a surface wave. Accordingly, it doesn't satisfy the orthogonality relation (14), but only relation (11)

Zusammenfassen (Conclusions)

It was shown that Zenneck's surface wave is physically realizable, and that the problem of the excitation of this wave is largely analogous to an open waveguide, e.g. a two-wire transmission line. Orthogonality relations exist between surface waves and space waves, which permit the computation of the amplitude of the surface waves in a simple manner.

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Georg Goubau received his Ph.D. in 1931 under Professor Jonathan Zenneck at Munich Technical Institute. At the close of WWII, Dr. Goubau was taken to the US (Fort Monmouth, NJ) as part of "Operation Paperclip". He is well known for his work in electrically small antennas, and is the inventor of the Goubau transmission line and the Goubau antenna.



Georg Goubau (A'49) was born in Munich, Germany, on November 29, 1906. He received the Dipl. Phys. degree in 1930, and



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the Dr. Ing. degree in 1931, both from the Munich Technical University. From 1931 to 1939 he was employed in research and teaching in the physics department of the same university, under Professor Zenneck. During this time he was principally concerned with ionospheric in-

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In 1939 Dr. Goubau was appointed professor and director of the department of applied physics at the Friedrich-Schiller University, in Jena, Germany. Before he arrived in this country, he was the senior author of the volumes on electronics of the FIAT Review of German Science, published by the Military Government for Germany. Dr. Goubau is now a consultant at the Signal Corps Engineering Laboratories, in Fort Monmouth, N. J.



(Proceedings of the IRE, Vol. 40, July 1952, p. 841.)

Comments

The analytical details of Goubau's cylindrical coordinate derivation are carried out in:

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Goubau's (integral) orthogonality relation between surface waves and radiation fields is carried out and discussed in:

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