THE SURFACE WAVE IN RADIO PROPAGATION OVER PLANE EARTH*

BY

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Summary—The results of Weyl for radio propagation over plane earth are found to differ from those of Sommerfeld by exactly Sommerfeld’s surface wave. Experiments conducted under conditions for which these two theories differ greatly are entirely consistent with Weyl’s results and show that Sommerfeld’s surface wave is not set up by simple antennas. Accordingly the Sommerfeld-Rolf curves are in error for all conditions for which the dielectric constant cannot be neglected.

INTRODUCTION

In 1907 Zenneck1 showed that a plane interface between two semi-infinite media such as the ground and air could support an electromagnetic wave which is exponentially attenuated in the direction of propagation along the surface and vertically upwards and downwards from the interface. Zenneck did not show that an antenna could generate such a wave, but because this “surface wave” seemed to be a plausible explanation of the propagation of radio waves to great distances, it was accepted.

When Sommerfeld2 obtained a solution of the radiation from an antenna on the surface of the earth that contained a cylindrical surface wave which at great distances is analogous to the Zenneck wave, the case for the Zenneck wave seemed complete.

Later Weyl3 approaching the problem in a different manner obtained a solution which did not explicitly contain this surface wave component. It appears, however, that he was of the opinion that his result was numerically equivalent to that of Sommerfeld. Van der Pol and Niessen4 have obtained various expressions for the solution of this

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K. F. Niessen, “Über die entfernten Raumwellen eines vertikalen Dipol-
problem, which have apparently been considered mathematically equivalent to those of Sommerfeld. The curves calculated by Sommerfeld for the variation of the field with distance have been extended by Rolf so as to cover all practical ground conditions. These curves have been accepted except for minor criticisms of the approximations that necessarily had to be introduced in order to reduce the number of variables.

This apparently satisfactory state of the theory of radio propagation endured without question until recently. It is true that the experimental checks were not always good but this was considered a result of the inhomogeneities of the earth and irregularities of its surface. Whenever experimental values differed greatly with this theory it was under conditions for which the effect of curvature of the earth or the effect of the ionosphere could not be neglected.

Recently Norton has pointed out that the Sommerfeld formula from which the Rolf curves were derived, does not agree with more recent formulas of van der Pol and Niessen and of Sommerfeld and he has published an empirical formula which does not agree with the Sommerfeld-Rolf curves. The writer has pointed out that the results of Weyl are not in agreement with those of Sommerfeld and Rolf.


Arnold Sommerfeld, "Über die Ausbreitung der Wellen in der drahtlosen Telegraphie," Ann. der Phys., ser. 4, vol. 81, pp. 1135–1153; December 11, (1926). This paper is free from the error of Sommerfeld's earlier papers.

but actually differ from them by exactly this surface wave in question. He also has found that Norton’s results agree with those of Weyl. Since Norton has derived his results from a formula of van der Pol and Niessen, their formulas presumably agree with those of Weyl.

As a result of the realization that the mathematics contained an ambiguity, the writer on September 23, 1933, attempted to decide the question experimentally by measurements at Budd Lake, New Jersey, employing ultra-short waves. The results indicated that the water was too shallow to meet the requirements of the experiment, since the transmission resembled that over land instead of over fresh water. At that time an experiment over deep fresh water was planned which has recently been successfully carried out. The results prove conclusively that simple antennas do not generate a Sommerfeld surface wave. This is in agreement with recent theoretical work by Wise\(^1\) and Rice.\(^2\)

As an immediate practical consequence of this work we are able to say definitely that the attenuation curves given by Sommerfeld and Rolf are incorrect for all types of ground for which the dielectric constant cannot be neglected.

**Experiment**

When an attempt is made to determine which of the two formulas for radio propagation is correct, on the basis of previously available experimental data, several difficulties are encountered. First, the available data has been taken under conditions where the difference between the two formulas is small; second, the data have been taken under conditions where irregularities of the earth’s surface are sufficient to produce variations in the received field strength of this order of magnitude. Third, the ground constants are not known by independent measurements with a sufficient degree of accuracy. All of these difficulties may be removed by making measurements on ultra-short-wave propagation over fresh water. Here the two formulas predict field strengths that differ enormously. The irregularities of the earth’s surface are reduced to a minimum by making measurements over calm fresh water, the inhomogeneities are removed by employing deep water, and finally the dielectric constant of the water can be determined from its temperature and the conductivity by measurements in the laboratory.

Seneca Lake in New York state was chosen for these experiments\(^3\)

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because of its great depth. A wave length of two meters (150 mega-
cycles) was chosen as a convenient ultra-short wave since apparatus
was available for this wave length.

These experiments are divided into two parts; (1) a determination
of the variation of the received field strength with distance, and (2)
a determination of the variation of the received field strength with
antenna height at a distance for which the predictions of the two
formulas differ greatly.

Fig. 1—Experimental arrangement for determining the variation of the
received field strength with distance.

**Variation of Field Strength With Distance**

Fig. 1 shows a picture of the experimental arrangement for determin-
ing the variation of the received field strength with distance. The re-
ceiver was located near the stern of a small motorboat which towed a
rowboat containing the transmitter. The antennas were loaded vertical
quarter-wave doublets which were connected to the receiver and trans-
mitter respectively at their mid-points by short two-wire transmission
lines. This equipment was driven along path 1 of Fig. 2, the distance
between antennas being maintained constant long enough to make
certain that there was no variation in the received field strength such
as might be caused by the bottom of the lake if the water were not
sufficiently deep. The distance between the transmitting and receiving
antennas was measured by means of an auxiliary line. This consisted
of a fish line for which the shrinkage due to becoming wet, or stretch
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due to increased tension, was less than one per cent by experimental test. The distances at which measurements were made were laid off on this line when it was under a fixed tension. The same tension was applied to this line when making the measurements. The solid circles of Fig. 3 are a plot of the experimental data so obtained.

For distances greater than 150 meters it was necessary to change the experimental procedure slightly. The receiver was located at the end of a pier and the transmitter carried in the motorboat. The trans-

![Diagram of Seneca Lake](image)

**Fig. 2**—Map of part of Seneca Lake showing the location of the experiment. Path 1 shows the location of the two-boat experiment, path 2 the one-boat experiment. Locations 3 and 4 indicate the positions of the terminals for the variable height test.

mitter was driven over path 2 of Fig. 2. This method of procedure introduced difficulty in measuring the distances between transmitter and receiver. In an effort to reduce the error in measurement of distance, the distance was obtained by three independent methods. First, the motorboat was driven at a constant speed and a fixed direction across the lake between two points a known distance apart. Second, the distance was measured by a transit located on the receiving pier and a stadia rod erected on the motorboat. Third, the distance was measured by determining with a sextant the angle subtended at the boat by two poles erected on the shore, one at, and the other near the receiver. The angle between the line joining the two poles and the direction to the boat was also determined by means of the transit; the distance be-
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tween the poles was measured directly. For this part of the experiment a continuous record of the received field strength was obtained by means of a manually operated recorder attached to the receiver. The open circles shown in Fig. 3 represent a plot of the data so obtained.

The smooth curves were calculated by means of the following formulas, using values of $\epsilon$ and $\sigma$ determined as follows: During the period of test the surface water temperature was 15.3 degrees centigrade which gives $\epsilon = 82.1$. The conductivity of five samples of water taken from various parts of the lake along which measurements were made was determined by L. A. Wooton of these Laboratories. The average value was found to be $4.988 \times 10^8$ electrostatic units with mean deviation of $0.006 \times 10^8$ at 1000 cycles and 25 degrees. Taking into account the effect of temperature on the conductivity this gave $\sigma = 4.05 \times 10^8$ electrostatic units.

Fig. 3—Variation of received field strength with distance. Curve (1) is plot of equation (1) showing the inverse distance field that would result from propagation over plane earth of perfect conductivity. Curve (2) is a plot of equation (2) showing the variation of the received field strength according to Weyl and Norton. Curves (3) and (4) are plots of equation (3) showing the variation of the received field strength according to Sommerfeld and Rolf. Curves (2) and (3) are based on a dielectric constant of 82.1, a conductivity of $4.05 \times 10^8$ electrostatic units and a wave length of two meters. Curve (4) is for a perfect dielectric. The experimental points were obtained on a wave length of two meters with loaded quarter-wave doublets whose mid-points were 0.52 and 0.60 meter above the surface of the water. The solid circles represent measurements made with two boats and the open circles represent measurements made with one boat.

$^{12}$ See International Critical Tables, vol. 6, p. 78: $\epsilon = 80 - 0.4 (T - 20)$.

$^{13}$ Taking data from the International Critical Tables, vol. 6, pp. 233 and 234 the conductivity at 15.3 degrees would be 4.02 and 4.07 if the electrolyte were NaCl and KCl, respectively. A single measurement of the conductivity at 15 degrees centigrade $\pm \frac{1}{2}$ degree centigrade gave a conductivity of 3.96.
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Curve (1) is a plot of the received field strength that would result from transmission over a plane earth of perfect conductivity:

\[ E_0 = \frac{120\pi HI}{\lambda r} \]  

(1)

Curves 2 and 3 result from multiplying (1) by the magnitudes of

\[ W = A - B/2 \approx C \]  

(2)

and

\[ S = A + B/2 \approx C + B \]  

(3)

respectively, where,

\[ A = 1 + \sum_{n=1}^{\infty} \frac{x^n e^{2in(\delta - \pi/4)}}{1 \cdot 3 \cdot 5 \cdots (2n - 1)} \]  

(4)

\[ B = \sqrt{2\pi x} e^{-(x/2)} e^{i(x/2) \sin 2\delta} e^{2\delta + i(x/2) \cos 2\delta + 3\pi/4} \]  

(5)

\[ C = - \sum_{n=1}^{\infty} \frac{x^n e^{2in(\delta - \pi/4)}}{1 \cdot 3 \cdot 5 \cdots (2n - 1)} \]  

(6)

\[ xe^{2i\delta} = \frac{2\pi r/\lambda}{\epsilon - 2i\sigma/\lambda}, \quad 0 \leq \delta \leq \pi/4. \]  

(7)

These follow from expressions given by Wise\(^5\) when the magnitude of \(e^{-2i\sigma/\lambda}\) is large compared with unity.\(^6\) \(\mid W \mid\) is the attenuation factor corresponding to the formula derived by Weyl. \(\mid S \mid\) is the attenuation factor as derived by Sommerfeld and used by him and by Rolf to calculate the variation of field strength with distance. \(B\), the difference between \(S\) and \(W\), corresponds to the surface wave component. For a perfect dielectric the exponent in \(B\) is a pure imaginary and curve (3) becomes curve (4).

The experimental points are in good agreement with curve 2 which is a plot of (2) and agrees with Weyl and Norton. At distances less than five meters (2\(\frac{1}{2}\) wave lengths) the experimental points lie slightly below the theoretical curve and show a tendency towards oscillation. This is presumably due to the combined effect of the finite size of the antennas and their finite height above the water's surface. These oscillations may

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\(^6\) These expressions differ from those of Wise in that the sign of \(i\) has been changed so that the implied time factor is \(e^{t+\omega t}\) in accordance with engineering practice instead of the \(e^{-\omega t}\) employed by Sommerfeld and Wise. Equation (4) is equivalent to Wise's expression (5). Equation (5) follows from the negative of Wise's (12) when it is remembered that changing the sign of \(i\) changes the Hankel function of the first kind into the corresponding one of the second kind and the Hankel function is replaced by the first term of its asymptotic expansion. Equation (6) is equivalent to Wise's expression (8).
be a vestige of the pronounced interference pattern that extends to greater distances with higher antennas. The discrepancy between the experimental points and curve 3 which is a plot of Sommerfeld's formula is so great that there can be no doubt as to the incorrectness of the latter.

VARIATION OF FIELD STRENGTH WITH HEIGHT

To determine the variation of received field strength with antenna height, 25-meter portable masts were erected at locations 3 and 4 of Fig. 2. Fig. 4 shows a picture of the transmitting location at 4. Unfortunately it was impossible to get a location at the water’s edge sufficiently removed from trees and the cliffs along the side of the lake to remove the possibility of their affecting to some extent the received
field strength. On vertical polarization the received field strength was determined as a function of the receiving antenna height for transmitting antenna heights of 24.8 and 2.5 meters (above the surface of the water). The measurements were repeated on horizontal polarization for the larger antenna heights. These experimental results are compared with theory in Fig. 5.

For the conditions of this part of the experiment $B$ is large compared with $C$, so that $W \approx C$ and $S \approx B$. A more general expression for $|B|$ not limited to the surface of the earth, nor to large values of the complex index of refraction (but limited to $\epsilon > 2\sigma f$) is:

$$|B| = \frac{2\pi \epsilon^{\theta/\lambda}}{(\epsilon + 1)^{\theta/\lambda} - 1} \sqrt{\frac{\pi}{\epsilon \lambda}} e^{\frac{2\pi}{\lambda} \frac{\sigma f}{\sqrt{\epsilon (\epsilon + 1)\lambda}}} - \frac{2\pi (h_1 + h_2)}{(\epsilon + 1)^{\theta/\lambda}} \frac{\sigma f}{\epsilon (\epsilon + 1)\lambda}. \quad (8)$$

In a companion paper it is shown that if there is no surface wave the attenuation factor is to a first approximation

For the conditions of the experiment the error introduced by the approximations of (8), (9), (10), and (11) is entirely negligible.

\[ \frac{E}{E_0} = \frac{1}{2} \left| 1 + \left( R + \frac{(R + 1)^2 \lambda r}{2\pi i(h_1 + h_2)^2} \right) e^{-i\phi_1, h_1}/\lambda r \right| \]  

(9)

where \( R \) is the appropriate coefficient of reflection. The smooth curves of Fig. 5 are plots of this expression for the conditions of the experiment.

Sommerfeld\(^{20}\) has shown that a horizontal electric antenna does not generate a surface wave in the direction perpendicular to the axis of the antenna. Hence there is no uncertainty as to the correct expression for the field in the case of horizontal antennas. This, together with the fact that at the greater heights the antennas may be considered to be in free space as far as their impedance is concerned, allows the measurements with horizontal polarization to be taken as a calibration of the measuring equipment. Using this calibration the received field strength is found to be \(-31.5\) decibels above the inverse distance field with antenna heights of 2.5 and 1.6 meters. For antennas on the ground Weyl’s formula gives a value of \(-36.8\) decibels while that of Sommerfeld gives value of \(+6.6\) decibels. Hence the absolute value of the received field strength is in agreement with Weyl.

These experimental results are compared with theory in Fig. 5. The measured value of field strength actually decreases with decrease in antenna height whereas Sommerfeld’s formula predicts an increase as shown by curves (4) and (5). The oscillations in the experimental curves are presumably due to reflections from the cliffs and trees behind the receiving antenna. While the agreement between the experimental and theoretical curves is not all that could be desired, it is the best that could be expected under the conditions of the experiment and in no way introduces any doubt as to the error in the Sommerfeld curves which predict a field strength about 100 times that measured.

\(^{19}\) When \( \epsilon > 2\sigma / f \), \( R = -k \) and if in addition \( h_1 + h_2 \) is small

\[
(1 - k) = \frac{2 \epsilon}{\sqrt{\epsilon - 1}} \frac{h_1 + h_2}{d}
\]

for vertical polarization and

\[
(1 - k) = \frac{2}{\sqrt{\epsilon - 1}} \frac{h_1 + h_2}{d}
\]

for horizontal polarization. Under these conditions (9) reduces to

\[
|C| = \frac{\epsilon^2}{(\epsilon - 1)(2\pi r/\lambda)} \left[ 1 + \frac{\epsilon - 1}{\epsilon^2} \left( \frac{2\pi h_1}{\lambda} \right)^2 \right] \left[ 1 + \frac{\epsilon - 1}{\epsilon^2} \left( \frac{2\pi h_2}{\lambda} \right)^2 \right]
\]

(10)

for vertical polarization and to

\[
|D| = \frac{2\pi h_1 h_2}{\lambda r} \left[ 1 + \frac{1}{\epsilon - 1} \left( \frac{\lambda}{2\pi h_2} \right)^2 \right] \left[ 1 + \frac{1}{\epsilon - 1} \left( \frac{\lambda}{2\pi h_2} \right)^2 \right]
\]

(11)

for horizontal polarization. The coefficient of the radical in (10) gives the value of \(|C|\) on the ground.

CONCLUSIONS

These experiments on the propagation of two-meter waves over Seneca Lake have shown that the surface wave component of Sommerfield is not set up by simple antennas on the surface of the earth. Measurements were made on the variations of the field strength with distance, under conditions for which this component, if present, would have been large compared with all the other components. They agreed with the curves calculated by neglecting this component. Measurements made on the variation of the received field strength with antenna height under similar conditions showed that the received field strength increased with antenna height, whereas Sommerfeld's surface wave would have decreased. Finally the absolute value of the received field strength was found to be less than that predicted by Sommerfeld by a factor of about a hundred. Accordingly the Sommerfeld-Rolf curves are in error for all conditions for which the dielectric constant cannot be neglected.

As a result of this fact it follows that the asymptotic series development of the received field strength is a true asymptotic expansion and does not require the addition of an exponential term. Hence the series development of the problem by Strutt$^{21}$ and Wise$^{22,23}$ may be used with confidence.

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$^{22}$ W. Howard Wise, "Note on dipole radiation theory," Physics, vol. 4, pp. 354-358; October, (1933).